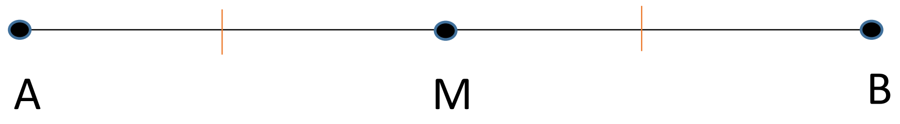
Chapter 1

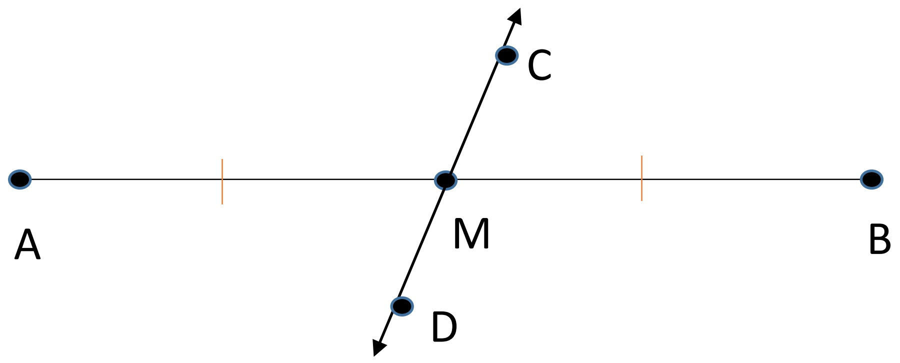
Basics of Geometry

Section 5
Segment and Angle Bisectors

The __midpoint__ of a segment is the point that divides, or __bisects__, the segment into two congruent segments. In this book, matched red *congruence marks* identify congruent segments in diagrams.

A __segment bisector__ is a segment, ray, line, or plane that intersects a segment at its midpoint.





M is the midpoint of $\overline{AB}$ if
M is on $\overline{AB}$ and AM = MB.

$\overleftrightarrow{CD}$ is a bisector of $\overline{AB}$.

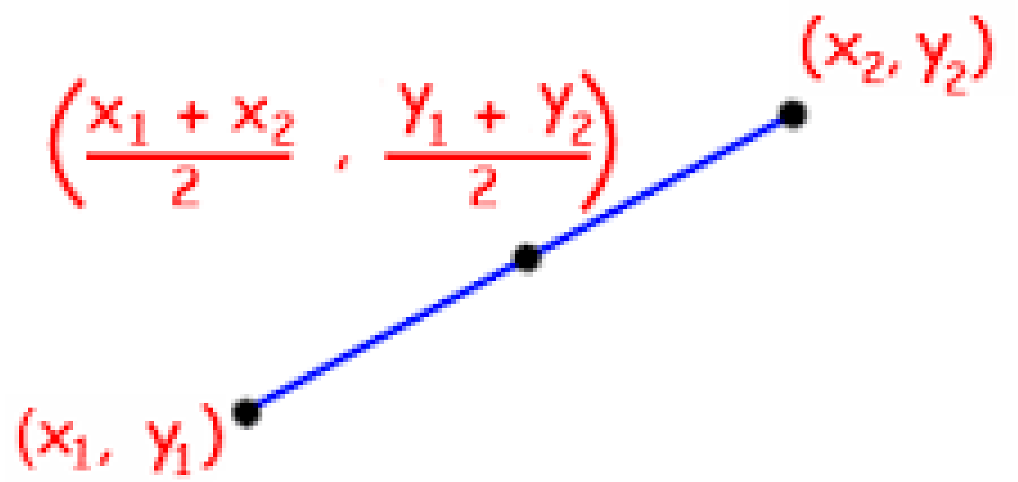
You can use a compass and a straightedge (a ruler without marks) to construct a segment bisector and midpoint of $\overline{AB}$. A construction is a geometric drawing that uses a limited set of tools, usually a compass and a straightedge.

If you know the coordinates of the endpoints of a segment, you can calculate the coordinates of the midpoint. You simply take the mean, or average, of the x-coordinates and of the y-coordinates. This methods is summarized as the Midpoint Formula.

The Midpoint Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint of AB has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Example 1: Finding the Coordinates of the Midpoint of a Segment

Find the coordinates of the midpoint of $\overline{AB}$ with endpoints

A(-2, 3) and B(5, -2).

$$\left(\frac{-2+5}{2}, \frac{3+{-2}}{2}\right)$$

$$\left(\frac{3}{2}, \frac{1}{2}\right)$$

$$(1.5, .5)$$

Example 2: Finding the Coordinates of an Endpoint of a Segment

TEST

The midpoint of $\overline{RP}$ is M(2, 4). One endpoint is R(-1, 7). Find the coordinates of the other endpoint.

$$\left(\frac{x+-1}{2}, \frac{y+7}{2}\right) = (2, 4)$$

$$2 \times \frac{x+-1}{2} = 2 \times 2 \qquad 2 \times \frac{y+7}{2} = 4 \times 2$$

$$x - 1 = 4 \qquad y + 7 = 8$$

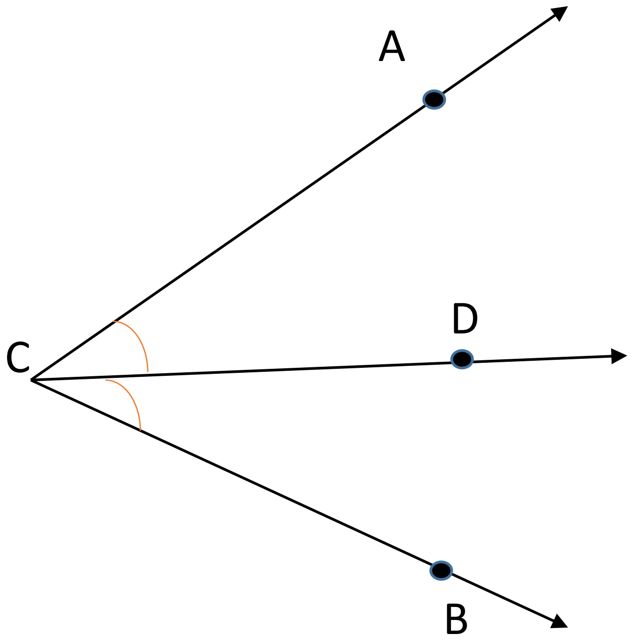
$$+1 \quad +1 \qquad -7 \quad -7$$

$$x = 5 \qquad y = 1$$

$$\boxed{(5, 1)}$$

GOAL 2: Bisecting an Angle

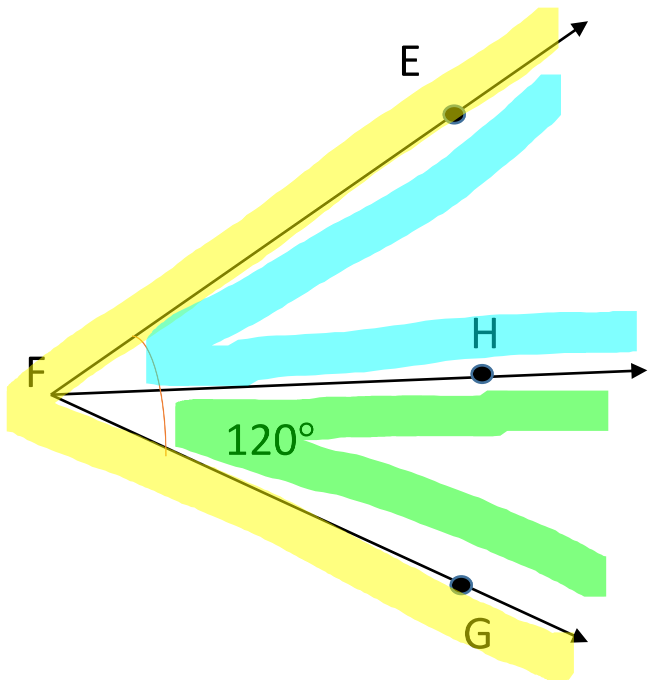
An angle bisector is a ray that divides an angle into two adjacent angles that are congruent. In the diagram at the right, the ray CD bisects <ABC because it divides the angle into two congruent angles, <ACD and <BCD.



In this book, matching *congruence arcs* identify congruent angles in diagrams.

Example 3: Dividing an Angle Measure in Half

The ray $\overrightarrow{FH}$ bisects the angle <EFG. Given that m<EFG = 120°, what are the measures of <EFH and <HFG?



120/2 = 60

m<EFH = 60*

m<HFG = 60*

Example 4: Doubling an Angle Measure

In the kite, two angles are bisected.

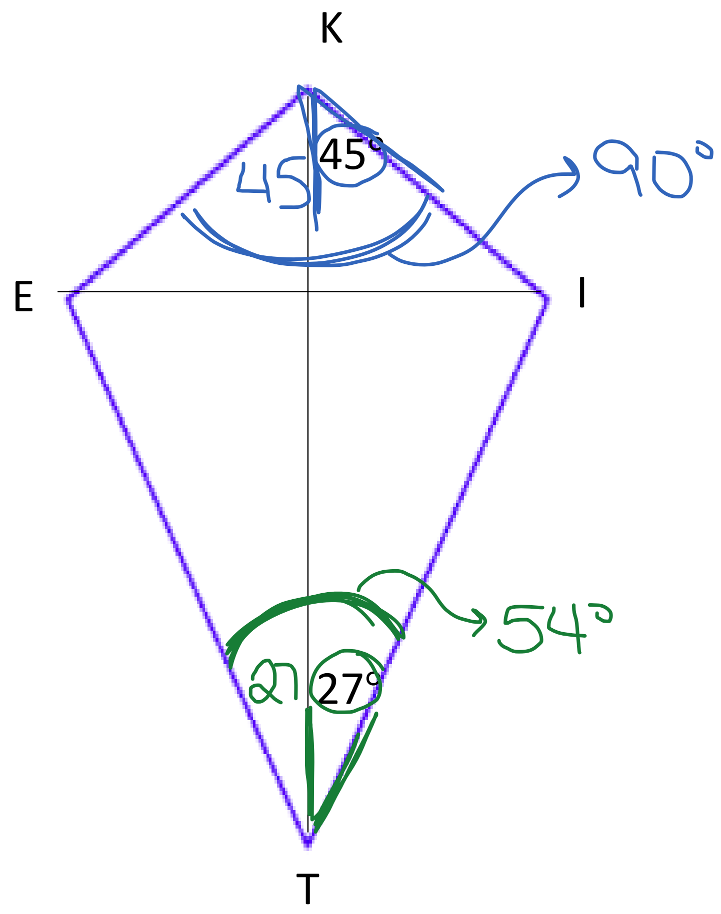
<EKI is bisected by KT.

<ITE is bisected by TK.

Find the measure of the two angles.

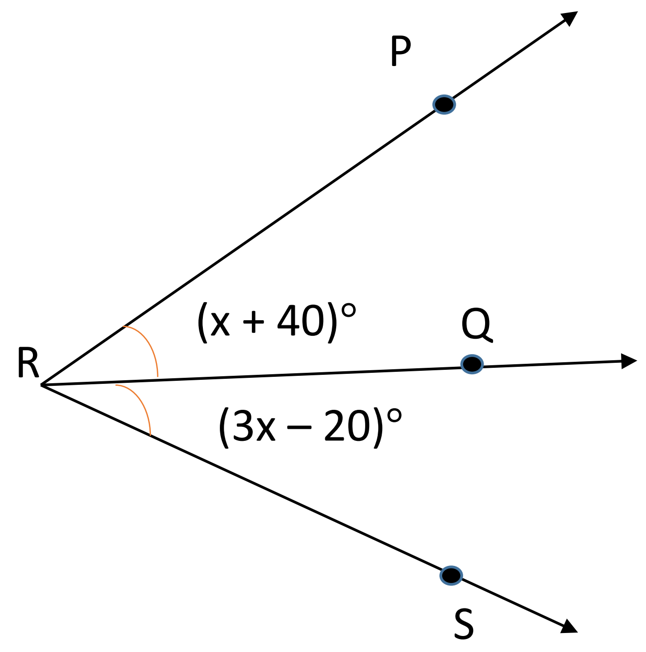
m<EKI = 90*   (45 + 45)

m<ETI = 54*   (27 + 27)



Example 5: Finding the Measure of an Angle

In the diagram, RQ bisect <PRS. The measures of the two congruent angles are (x + 40)° and (3x – 20)°. Solve for x.



$$\cancel{x} + 40 = 3x - 20$$
$$-\cancel{x} \qquad\qquad -x$$
$$40 = 2x - \cancel{20}$$
$$+20 \qquad\qquad +\cancel{20}$$
$$\frac{60}{2} = \frac{\cancel{2}x}{\cancel{2}}$$
$$30 = x$$